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Upscaling Relative Phase Permeability for Superelement Modeling of Petroleum Reservoir Engineering

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Abstract—A technique for locally rescaling (upscaling) the functions of the relative phase permeability (RPP) has been developed, which minimizes the error in the approximating the phase filtration rates for the superelement modeling of waterflooding a layered heterogeneous oil reservoir. The RPP is locally upscaled for each superelement based on the solution of two-dimensional two-phase filtration problems on a refined computational grid. Modified RPP functions (MFRPPs) are represented in the parametric form; i.e., the values of the parameters are sought when solving the problem of minimizing the deviations of the averaged and approximated phase velocities at the sites corresponding to the faces of the superelement. The efficiency of applying MFRPP to superelement modeling is illustrated by an example of a model reservoir region where oil is extracted using injection and production wells and by an example of waterflooding at a real oilfield.

Keywords: upscaling, relative phase permeability, two-phase filtration, modeling of petroleum reservoir engineering, superelements

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1. INTRODUCTION

Optimizing the waterflooding of oil reservoirs involves carrying out multivariate adaptation and forecast calculations for the filtration model. Most such models are mathematically based on two nonlinear differential equations, i.e., a parabolic equation for the pressure p and a hyperbolic equation for the water saturation s. In the Cartesian coordinate system $x, y, z,$ and t with the vertical axis z in dimensionless variables they have the following form (see, e.g., [1])

$$\beta\frac{\partial p}{\partial t} + \nabla \mathbf{u} = 0, \quad \mathbf{u} = -\sigma(s)\nabla p; \tag{1}$$

$$\frac{\partial (ms)}{\partial t} + \nabla\left(f(s)\mathbf{u}\right) = 0. \tag{2}$$

Here, β is the elastic capacity, $\mathbf{u}$ denotes the filtration rate of a two-phase fluid, and m is the reservoir porosity. Functions $\sigma(s)$ and $f(s)$ describe hydroconductivity and water fraction in the two-phase flow and are obtained by the formulas

$$\sigma = k\varphi, \quad \varphi(s) = k_w(s) + \mathrm{K}_\mu k_o(s), \tag{3}$$

$$f(s) = k_w(s)/\varphi(s), \quad \mathrm{K}_\mu = \mu_w/\mu_0, \tag{4}$$

where k is the absolute permeability of the reservoir, $\mu_{w,o}$ denotes the dynamic viscosity of aqueous (w) and oil (o) phases, and $k_{w,o}$ designates the relative phase permeabilities (RPPs), which are approximated by the power-law dependences

$$k_w(s) = s^a, \quad k_o(s) = (1-s)^b, \quad s \in [0,1], \quad a, b = 1\text{–}4. \tag{5}$$

Equations (1)–(5) describe two-phase filtration in the area in the domain D bounded from above by the top of the reservoir $z = T(x, y)$; from below, by the base surface $z = B(x, y)$; and on the side, by the outer cylindrical surface Γ, as well as by the inner boundaries $\gamma_i, i = 1, \ldots, N_W$—the surfaces of the perforated sections of the wells—of pipes with a diameter of r_w. On the outer sections of the boundaries, the